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A digital method for sinewave generation

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Contents: A simple method to digitally produce a sine waveform with constant amplitude over a relatively wide band of frequencies is described. The method is based on the generation of a number of samples per cycle of the sine waveform, using an up/down counter and a decoder, and then processing these samples in a summing amplifier and in a controlled-gain amplifier to produce a staircase approximation of the sinewave. The total harmonic distortion of the resulting waveform can be reduced substantially by inserting a low-pass filter.

Methode zur digitalen Erzeugung von Sinuswellen

Übersicht: Es wird ein einfaches Verfahren zur Erzeugung von Sinuswellen mit konstanter Amplitude über ein relativ breites Frequenzband beschrieben. Das Verfahren beruht auf der Erzeugung einer Anzahl von Stichproben innerhalb einer Sinuswellenperiode, indem ein Aufwärts-/Abwärts-Zähler und ein Dekodierer herangezogen werden, um daraufhin diese Stichproben in einem Addierer und einem Verstärker mit geregelter Verstärkung zu verarbeiten und dadurch eine stufenartige Näherung der Sinuswelle zu erreichen. Die Gesamtverformung der Oberschwingungen (THD) der entstandenen Sinuswelle kann durch ein Tiefpaß-Filter herabgesetzt werden.

1 Introduction

A number of methods have been reported in recent years for generating sinusoid and other waveforms, using digital techniques. These methods include storage of digital samples in Read-Only Memories and then extracting the information with D/A converters [1–5], filtering a square wave with a switched-capacitor filter [6] and waveform synthesis using Walsh functions [7–8]. All these methods provide circuits of various degrees of complexity and cost. Digital generation of analog waveforms is now gaining practical significance in many applications involving microprocessor-controlled systems.

This paper describes a relatively simple method to digitally generate sinusoidal waveforms of constant amplitude over a relatively wide band of frequencies. A prototype has been built and evaluated for use in an Applied Potential Tomography system.

2 Method description

Figure 1 is a block diagram showing the steps followed in the process of digitally generating a symmetric waveform. The clock provides pulses to an up/down counter which drives an m-line-to-n-line decoder. The up/down counter applies to the m lines an input address which is decoded as a positive pulse that appears at only one of the n output lines, while the rest of the output lines are at zero logic level. Each of the decoder output pulses is applied, one at a time, to a summing amplifier whose input resistances are chosen such that samples having the appropriate amplitude for a portion of the desired waveform will be obtained at the output of the summing amplifier. This waveform portion is finally applied to a controlled-gain amplifier whose voltage gain is switched from $+1$ to -1 at

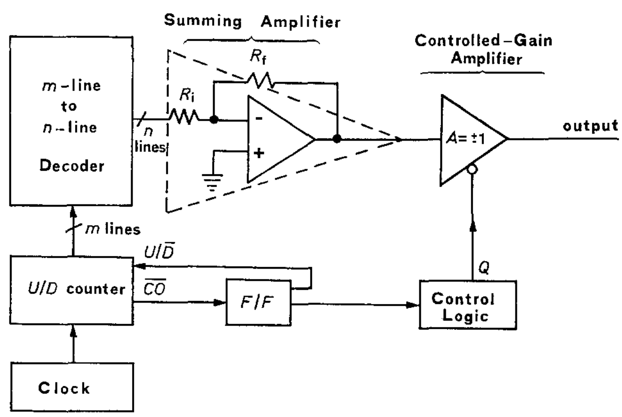

Fig. 1. Block Diagram for sinewave generation

the correct times to provide at its output an approximation of the desired waveform. From the above, it should be evident that for the desired waveform obtained at the amplifier output: a) the purity will depend on the number N of samples per cycle used to generate it, b) the frequency will be equal to the frequency of the clock pulses divided by N and c) the amplitude will be influenced only by the amplitude frequency characteristics of the summing amplifier and the controlled-gain amplifier.

3 Generation of sine waveforms

The implementation of the method described above will be given now for the case of sine waveform generation, but it will become clear that the method can be used also for the generation of other types of symmetric waveforms.

A sine waveform can be approximated by N equally-spaced samples per cycle, as shown in Fig. 2. Each sample has a time duration τ, equal to the period of the clock pulses, and an amplitude which may be expressed as $\sin(2\pi k/N)$, where $k = 0, 1, 2, 3, \ldots$ As a consequence, the sine waveform period is $T_0 = N\tau$ and the digitally-produced sine function is given by

$$f(t) = \sin\left(\frac{2\pi}{N\tau}\right) t = \sin\left(\frac{2\pi k}{N}\right) \tag{1}$$

where t takes only the values given by $t = k\tau$ and $k = 0, 1, 2, 3, \ldots$

It should be noted that, because of symmetry, only the positive cycle of the sine waveform needs to be generated repeatedly by the decoder-summing-amplifier section of the circuitry. The negative cycle can be obtained by inverting the positive cycle every other time. This is done as shown in Fig. 1 by the controlled-gain amplifier whose voltage gain is set $+1$ or -1, alternately, by the control logic whose timing is synchronized by the up/down counter circuitry. As a consequence of the sinewave symmetry around the $y = \pi/2$ axis, only $N/4$ resistors are needed at the summing amplifier inputs. The waveforms at the terminals of interest of the up/down counter,

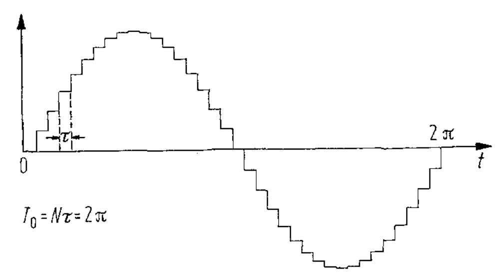

Fig. 2. A staircase sinewave approximation

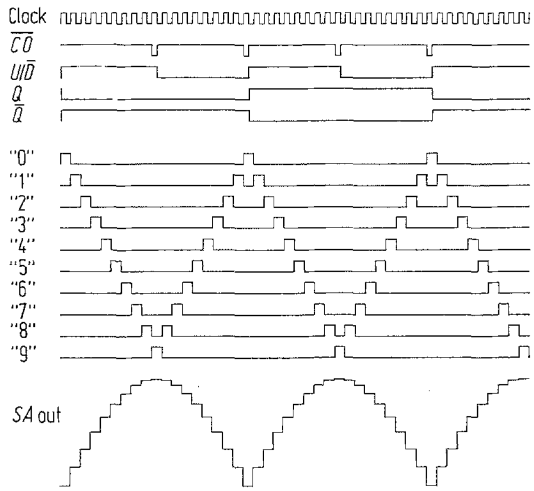

Fig. 3. Basic waveforms for the formation of a staircase sinewave portion

the decoder and the summing amplifier for the case that $N = 36$ are shown in Fig. 3.

The clock pulses are applied to an up/down counter (CD4029) which drives a BCD-to-decimal decoder (CD4028). It is noted in Fig. 3 that each clock pulse produces a single positive pulse at one of the decoder output lines selected by the decoder BCD input address. Each of these pulses, whose amplitude is constant and whose time duration is equal to the clock pulse period, is applied to the summing amplifier through a resistance chosen such that the resulting input current will produce the appropriate amplitude of the sinewave sample. It is also noted that the tenth clock pulse (from the beginning of each positive cycle) produces the peak sinewave sample, and after a small delay, it places — for a short time — the carry out ($\overline{\text{CO}}$) signal of the up/down counter from the normally logical "1" state to the logical "0" state. The positively-going edge of this $\overline{\text{CO}}$ short pulse resets a F/F which thus applies a logical "0" level to the $UP/\overline{DN}$ input of the up/down counter and forces it to count down. From here on, the decoder keeps producing pulses that correspond to the downward portion of the sine waveform. After counting down nine pulses, the $\overline{\text{CO}}$ signal goes to logical "0" state again and its positively-going edge sets the F/F which places now the up/down counter to the count-up mode. This procedure is repeated, thus producing at the summing amplifier output the positive cycle of a sinewave approximated by a staircase having steps of variable amplitude. In order to obtain a full cycle of the sine waveform, a controlled-gain amplifier is used which has a voltage gain equal to $+1$ for the duration of one positive cycle and -1 for the next positive cycle and so on, resulting in the staircase waveform shown in Fig. 2.

4 Spectral analysis of the approximated sinewave

By looking at the sine waveform obtained at the controlled-gain amplifier output, it is evident — as mentioned previously — that the sinewave purity will depend on the number N of digital samples composing the waveform. In this section, we will examine that dependence by calculating the harmonic coefficients and the total harmonic distortion as functions of N.

In the Fourier analysis that follows, the sinewave is shifted by 90° in order to take advantage of the quarter-wave symmetry provided by the cosine function, as shown in Fig. 4. Thus, the staircase approximated cosine function is defined in the time interval $0 \leqq t < \frac{\pi}{2}$ as follows:

$$\left.\begin{aligned} &g(t) = 1 \quad \text{for} \quad 0 \leqq t < \frac{\tau}{2} \\ &\text{and} \\ &g(t) = \cos\left(\frac{2\pi\nu}{N}\right) \quad \text{for} \quad (2\nu - 1)\frac{\tau}{2} \leqq \\ &\qquad \leqq t < (2\nu + 1)\frac{\tau}{2} \end{aligned}\right\} \tag{2}$$

where

$$\nu = 1, 2, 3, \ldots \left(\frac{N}{4} - 1\right) \quad \text{and} \quad \tau = 2\pi/N$$

Consequently, the Fourier coefficient for the mth harmonic $|C_m|$ is given by

$$|C_m| = \frac{2}{m\pi}\left[\sin\frac{m\pi}{N} + \sum_{\nu=1}^{(N/4)-1} \cos\frac{2\pi\nu}{N} \times \left\{\sin(2\nu + 1)\frac{m\pi}{N} - \sin(2\nu - 1)\frac{m\pi}{N}\right\}\right] \tag{3}$$

where $m = kN \pm 1$ and $k = 0, 1, 2, 3, \ldots$

Table 1 shows the calculated harmonic coefficients for the case of $N = 36$ samples per cycle. The total harmonic distortion (THD) can be calculated from the defining expression:

$$\text{THD} = \left[\frac{\sum_{m=2}^{\infty} |C_m|^2}{\sum_{m=1}^{\infty} |C_m|^2}\right]^{1/2} \times 100\% \tag{4}$$

which can be written as

$$\left[1 - \frac{|C_1|^2}{\sum_{m=1}^{\infty} |C_m|^2}\right]^{1/2} \tag{5}$$

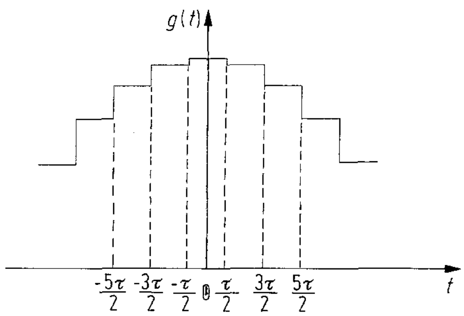

Fig. 4. Approximated staircase cosine function

Table 1. Example of calculated harmonic coefficients

Harmonic	Coefficient	Value for $N = 36$
Fundamental	$\lvert C_1 \rvert$	0.4993657
35th	$\lvert C_{35} \rvert$	0.0142676
37th	$\lvert C_{37} \rvert$	0.0134964
71st	$\lvert C_{71} \rvert$	0.0070333
73rd	$\lvert C_{73} \rvert$	0.0068406
107th	$\lvert C_{107} \rvert$	0.0046669
109th	$\lvert C_{109} \rvert$	0.0045813
143th	$\lvert C_{143} \rvert$	0.0034921
145th	$\lvert C_{145} \rvert$	0.0034439

From Table 2 which shows the calculated THD for several different values of samples per cycle, it is seen, as expected, that THD decreases as N increases. However, even for relatively large values of N (e.g. $N = 60$) the THD is still quite high and not suitable for some applications. In these cases, the THD can be reduced by inserting a low-pass filter (LPF) at the amplifier output having a cutoff frequency chosen such that the sinewave fundamental will not be affected substantially, while the next harmonic will be affected as much as possible. One way to satisfy this criterion is to choose the cutoff frequency four times higher than the fundamental, in which case (with $N = 36$) the fundamental amplitude is reduced by 3% (0.26 dB), while the next (35th) harmonic is reduced by 880% (18.89 dB). For that case, the THD of the staircase approximation of the sinewave goes from 5.02% to 0.46%. To reduce this even further, a second stage of the same type of filter can be added which brings the THD down to 0.05%. The THD for these three cases as a function of N is shown in Table 2.

Table 2. THD as a function of N

% THD	Number of samples per cycle, N									
	12	20	28	36	44	52	60	68	76	84
No LPF	15.00	9.02	6.45	5.02	4.10	3.47	3.01	2.65	2.37	2.15
One LPF	4.12	1.50	0.77	0.46	0.31	0.22	0.16	0.13	0.10	0.08
Two LPF	1.34	0.29	0.10	0.05	0.02	0.00	0.00	0.00	0.00	0.00

5 Experimental results

A prototype system was built to check the theoretical results of the previous section for the case that $N = 36$ samples per cycle. The number N can be easily increased to 60, simply a) by using the CD4029 chip as a binary up/down counter (by applying logical "1" at its binary/decade input), b) by replacing the CD4028 BCD-to-decimal decoder with a 4-bit binary to 16-bit decoder (such as 74C154) and c) by appropriately modifying the control logic circuit in Fig. 1.

Figure 5 shows an oscilloscope photograph of the staircase approximation of a sine waveform (upper trace) and of the same waveform (lower trace) after passing through a LPF having a cutoff frequency equal to four times the sinewave frequency. The amplitudes of the fundamental and of several harmonics for the staircase approximation are shown in Fig. 6, where it is noted that the harmonic amplitude decreases slowly as the order of the harmonic increases.

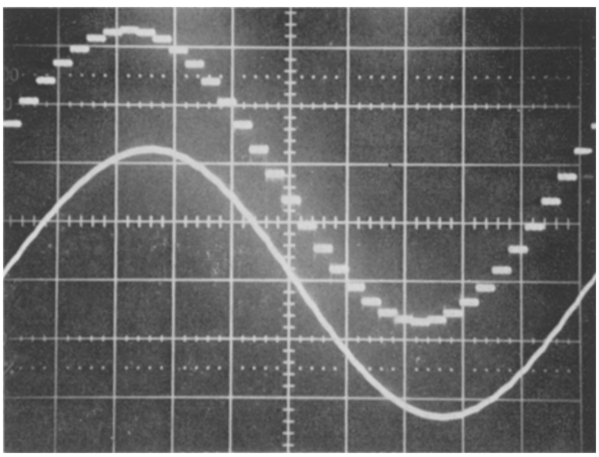

Fig. 5. Experimental staircase approximation (upper trace) and filtered sine waveform (lower trace)

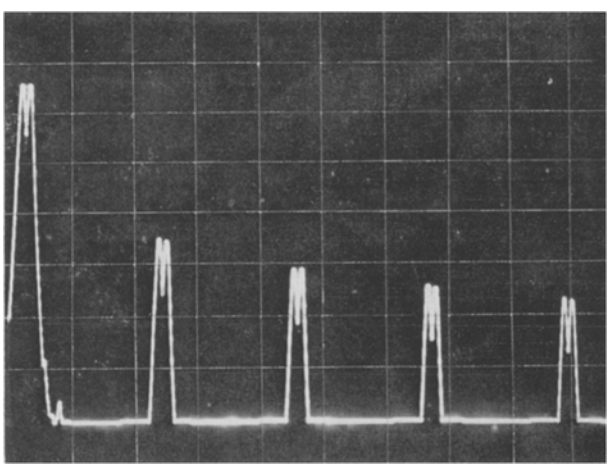

Fig. 6. Spectrum of a staircase approximated sinewave. Vertical axis scale: 10 dB/div

Table 3. Comparison between theoretical and experimental relative harmonic levels in dB for $N = 36$

Harmonic	No LPF		One LPF		Two LPF	
	Theory	Exp.	Theory	Exp.	Theory	Exp.
35th	−30.88	−29	−49.78	−49	−68.67	−68
37th	−31.68	−30	−51.05	−49	−70.43	−68
71th	−37.03	−37	−62.00	−60	−87.02	<-80
73rd	−37.27	−37	−62.50	−60	−87.75	<-80
107th	−40.59	−40	−69.14	−69	−97.70	<-80
109th	−40.75	−40	−69.46	−69	−98.18	<-80
143rd	−43.11	−43	−74.18	−74	−105.25	<-80
145th	−43.23	−43	−74.42	−74	−105.61	<-80

Table 3 shows in more detail the relative levels of several harmonics with respect to the fundamental for $N = 36$ and for a) no LPF b) one LPF section and c) two LPF sections.

6 Conclusions

A relatively simple method has been described for digitally producing a sine waveform with constant amplitude over a wide band of frequencies. The resulting output is a staircase approximation of a sinewave whose purity has been studied as a function of the number N of samples per cycle composing the sinewave. It has been found that the relative amplitude of the harmonic coefficients decreases slowly with the harmonic order. Also, the total harmonic distortion which ranges between 9% for $N = 20$ to 2.15% for $N = 84$, is relatively high, even for large values of N, and unacceptable in many applications. It was found, however, that by inserting a simple, one-pole RC low-pass filter, having a cutoff frequency equal to four times the sinewave frequency, the total harmonic distortion is brought down in the range from 1.5% for $N = 20$ to 0.08% for $N = 84$. If a second filter of the same type is used, the total harmonic distortion is reduced now in the range from 0.30% for $N = 20$ to 0.053% for $N = 44$ and 0.0% for $N > 44$.

A prototype has been built to evaluate the method and excellent agreement has been found between theoretical and experimental values. The amplitude of the output waveform was found to remain constant for frequencies up to 12 kHz (for $N = 36$) corresponding to clock pulse frequency of 432 kHz. This limitation comes mainly from the CMOS ICs used in the prototype circuit and can be alleviated by using a faster IC family. Furthermore, it has been found that the output amplitude does not vary with temperature, because any resistance variations caused by temperature, are compensated for in the gain ratio of the feedback to the input resistances of the summing amplifier.

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